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# ADAPTATION OF A TURBULENT BOUNDARY LAYER ON A CONVEX SURFACE

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*The paper presents the results of numerical investigation of the adaptation of velocity profiles, turbulent shear stresses, and coefficients of friction in boundary-layer development along a convex surface and on transition of the boundary layer from a plane surface to convex one. A parameter is obtained that characterizes the completion of adaptation and its dependence on the basic flow parameters. Equations are presented for calculating coefficients of friction in the portion of boundary-layer adaptation and in the main portion.*

Curvilinear surfaces are typical of the elements of power plants and units. Coefficients of friction and heat transfer are changed and velocity and temperature profiles are deformed under the effect of curvature. Account for the influence of surface curvature on the boundary layer makes it possible to improve the accuracy of calculations of heat and hydrodynamic characteristics. At present, the main specific features of boundary layers on convex and concave surfaces have been studied. The most important problems are systematized in [1]. In particular, decrease in velocity profile fullness, depression in the coefficients of friction and heat transfer, and reduction of the boundary-layer thickness compared to a plane surface are observed on a convex surface. The behavior of the boundary layer with the combined effect of surface curvature, longitudinal pressure gradient, and outer turbulence is studied. A boundary layer often reaches a curvilinear wall from a plane surface, where it is formed and developed, i.e., the boundary layer has a certain initial thickness $\delta_0$ at the point of transition to the convex surface. It is shown in [2] that on passing from the plane to the convex surface the boundary layer *is adapted*, i.e., its parameters change from those formed on the plane wall to those typical of the developed curvilinear boundary layer. It was assumed that the same process also takes place in the development of the boundary layer on the convex surface (without transition from the plane plate). Adaptation occurs over some *adaptation portion* whose length depends on various factors, including $\delta_0$, the radius of surface curvature $R_w$, and flow parameters. Adaptation is assumed to be completed when the velocity profile stops changing. Then the boundary layer passes to the main portion, where its development obeys the laws for a stabilized curvilinear boundary layer.

The process of boundary-layer adaptation is practically unstudied. The present paper deals with the study of boundary-layer adaptation on a convex surface, determination of the parameter characterizing the completion of the adaptation process, and also with the analysis of the effect of various factors on this parameter. Boundary-layer characteristics were studied for two cases, viz., with the development of the boundary layer on the convex surface from zero (zeroth initial length) and with the transition of the boundary layer from a plane surface ($R_w \to \infty$) to a convex surface of constant curvature ($R_w = \text{const}$) (finite value of the initial thickness).

The numerical study is based on the solution of the system of differential equations for an incompressible turbulent boundary layer and the model of eddy viscosity [3]:

$$u\frac{\partial u}{\partial x} + v\frac{\partial}{\partial y}\left[\left(1 + \frac{y}{R_w}\right)u\right] = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\left(1 + \frac{y}{R_w}\right)\frac{\partial \tau}{\partial y} + \frac{2\tau}{\rho \cdot R_w},$$

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$$\frac{1}{\rho}\frac{\partial p}{\partial y}=\frac{u^2}{R_w+y},\quad \frac{\partial u}{\partial x}+\frac{\partial}{\partial y}\left[\left(1+\frac{y}{R_w}\right)v\right]=0, \tag{1}$$

$$\tau=\tau_v\rho\varepsilon\left(\frac{\partial u}{\partial y}-\frac{u}{R_w+y}\right),$$

where $\varepsilon$ is found from [3],

$$\varepsilon=\varepsilon_0 f\,|f|\,\left|\frac{\partial u}{\partial y}-\frac{u}{R_w+y}\right|,$$

$f$ is the correction allowing for the effect of curvature on eddy viscosity [3],

$$f=l/l_0=\exp(-30\mathrm{Ri})-3.5\mathrm{Ri}^2+1.2\mathrm{Ri}+0.3b,$$

$$b=\begin{cases}0 & \text{when } \mathrm{Ri}=0,\\ 1 & \text{when } \mathrm{Ri}>0,\end{cases}$$

$\mathrm{Ri}=[2(u/R_w)]/(du/dy)$ is the Richardson number. Velocity distribution in the outer flow was assigned by the law $ur=\mathrm{const}$. In calculations, the velocity was presented in the form $u/u_p$, where $u_p=U_{pw}R_w/(R_w+y)$.

The boundary conditions were assigned in the following form:

$$y=0\left|\begin{array}{l}u=0,\\ v=0,\end{array}\right.\quad y=\delta\left|\;u=u_{p\delta},\quad \frac{\partial u}{\partial y}=\left(\frac{\partial u_p}{\partial y}\right)_\delta;\right.$$

the boundary layer thickness was taken to be equal to the coordinate $y$ of the point where $u=0.99u_p$.

To allow for the change in the boundary-layer structure with a sudden change in the surface radius on transition from the plane wall ($R_w\to\infty$) to the convex wall ($R_w$), the formula for an effective curvature radius $R_{ef}$ [2] was used:

$$R_{ef}=\frac{R_w}{1-\exp(-x/R_w)}.$$

The value of $R_{ef}$ calculated at each integration step was placed in the expression for the Richardson number Ri and then in the formula for eddy viscosity.

In a flow past the convex surface, the velocity at the outer edge of the boundary layer $u_{p\delta}$ is not constant even in the absence of the outer longitudinal pressure gradient. By virtue of this, the term $\partial p/\partial x$ in the equation for motion along the $x$ axis was determined by the relation

$$\frac{\partial p}{\partial x}=\frac{\partial p_e}{\partial x}-\rho\left(\frac{\partial}{\partial x}\int_y^\delta\frac{U_{pw}^2R_w^2}{(R_w+y)^3}\left(\frac{u}{u_p}\right)^2dy\right),$$

which can be obtained by integration of the equation for motion along the $y$ axis and by further differentiation of it with respect to $x$. This expression determines the quantity $\partial p/\partial x$ entering (1). In the absence of the outer longitudinal gradient, $\partial p_e/\partial x=0$.

This system of equations and boundary conditions was solved by the finite-difference method on a variable-step grid. The obtained mathematical model was tested computationally using the conditions of different physical experiments. The test results are given in [2, 3]. The turbulent boundary layer on the convex surface at $\delta^{**}/R_w=0-0.013$, $\delta_0^{**}/R_w=0.0012-0.088$, $\mathrm{Re}^{**}=500-9000$ was calculated by the described model. Calculations were performed for different values of $R_w$, $\delta_0$, and $U_0$. This paper presents the results of the studies of mean velocity profiles, turbulent shear stresses, and coefficient of friction.

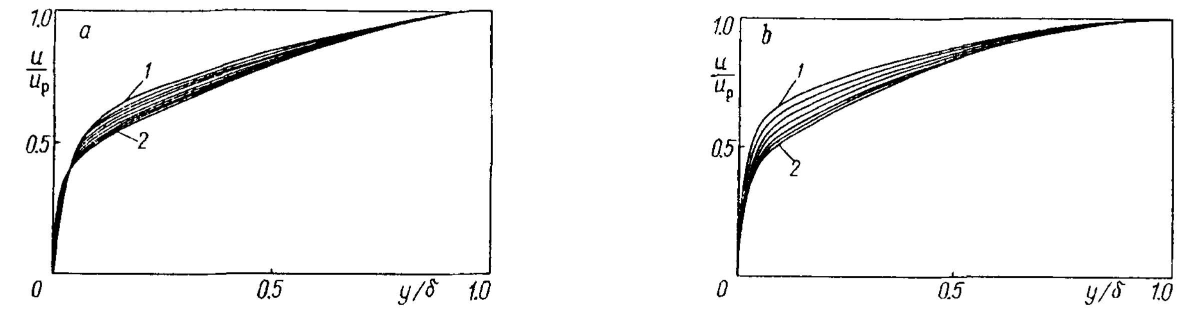



Fig. 1. Development of the velocity profile along a convex surface: a) $\delta_0 = 0$; 1) $\delta^{**}/R_w = 0.001$; 2) 0.0054; b) $\delta_0 \neq 0$; 1) plane surface at the transition point; 2) convex surface, $\delta^{**}/R_w = 0.006$; velocity profiles enclosed by profiles 1 and 2 correspond to intermediate values of the parameter $\delta^{**}/R_w$.

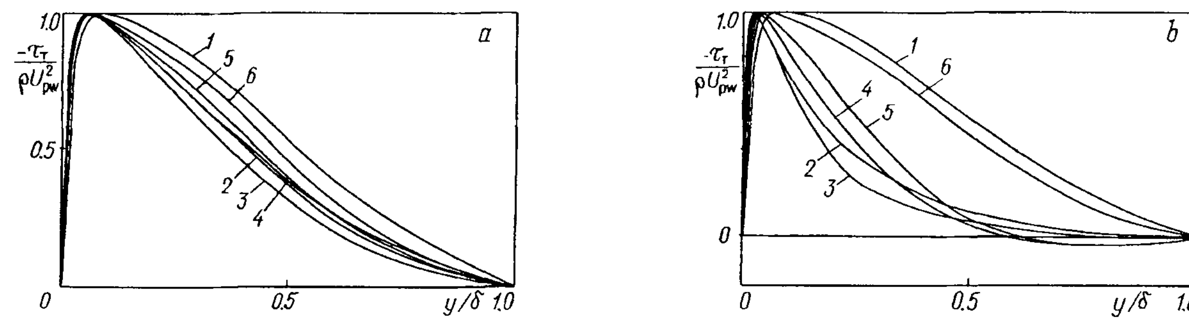


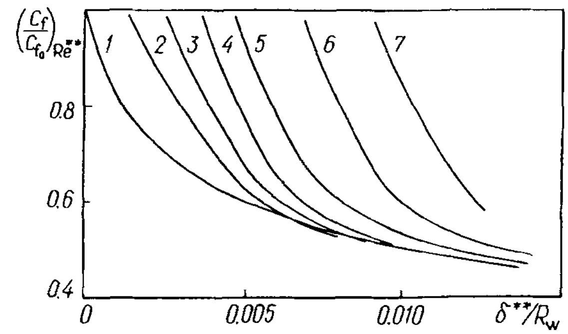

Fig. 2. Turbulent shear stresses for $\delta \neq 0$ [1) plane surface at the transition point; 2-5) formation of the profile after transition; 6) convex surface at $\delta_0 \neq 0$]: a) 1) $\delta_0^{**}/R_w = 0.0023$; 2) $\delta^{**}/R_w = 0.0026$; 3) 0.004; 4) 0.0054; 5, 6) 0.0068; b) 1) $\delta_0^{**}/R_w = 0.0096$; 2) $\delta^{**}/R_w = 0.01$; 3) 0.0109; 4) 0.0119; 5, 6) 0.0133.

With the development of the boundary layer near the convex surface (zeroth initial thickness) the fullness of the velocity profile in the outer region of the layer decreases under the effect of the curvature (Fig. 1a). In the internal region of the boundary layer, where the effect of viscosity is predominant, the fullness of the velocity profile increases. The analysis of the results obtained showed that in the case of $\delta_0 = 0$ adaptation of velocity profiles in the boundary layer takes place. The process of adaptation is nonlinear: the velocity of profile change decreases gradually and then the deformation of the profile stops, thus corresponding to cancellation of the process of adaptation. For all the calculations performed at $\delta_0 = 0$ and different values of $R_w$ and $U_0$, a constant value of the parameter $\delta_a^{**}/R_w$ was obtained, which characterizes the cancellation of the adaptation process and is equal to 0.0035.

Deformation of the profiles of turbulent shear stresses in the case of zeroth initial thickness of the boundary layer is characterized by the shift of the maximum of $\bar{\tau}_t$ toward smaller values of $y/\delta$. In the outer region of the boundary layer the changes in $\bar{\tau}_t$ are small and stop quickly as $\delta^{**}/R_w$ increases, thus indicating cancellation of the adaptation of turbulent shear stresses.

On transition of the turbulent boundary layer from the plane to the convex surface (zeroth initial thickness), the adaptation of the velocity profile from the form typical of the plane surface to that typical of the curvilinear surface occurs (Fig. 1b). The fullness of the velocity profile decreases in both the inner and the outer regions of the boundary layer. As in the case of the zeroth initial thickness, the process of adaptation has a nonlinear character and at some value of $\delta_a^{**}/R_w$ the adaptation of the profile halts. The studies conducted showed that the parameter $\delta_a^{**}/R_w$ depends on the initial curvature parameter $\delta_0^{**}/R_w$.

Figure 2 shows the adaptation of turbulent shear stresses in the case of zeroth initial thickness of the boundary layer. On transition from the plane to the convex surface, $\bar{\tau}_t$ first sharply decreases (curves 1, 2). Then leveling of the profiles (curves 3-5) and their approach to the shape typical of the curvilinear boundary layer (curves



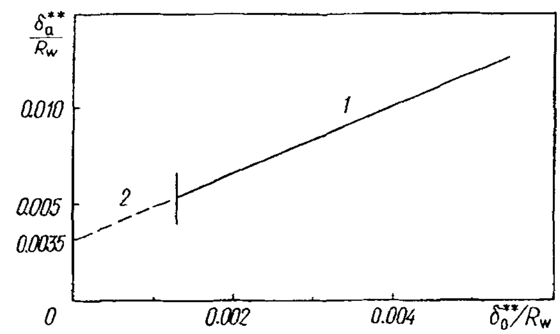




Fig. 3. Relative coefficient of friction in a boundary layer on a convex surface: 1) $\delta_0 = 0$; 2-7) $\delta_0 \neq 0$ [$\delta_0^{**}/R_w$: 2) 0.0012; 3) 0.0023; 4) 0.0035; 5) 0.0044; 6) 0.0066; 7) 0.0088 ].

Fig. 4. $\delta_a^{**}/R_w$ as a function of the initial curvature parameter: 1) calculation region; 2) extrapolation region.

6 in Fig. 2) are observed. The rate of initial decrease of $\tau_t$ grows with the initial parameter of curvature, which is confirmed by a comparative analysis of the profiles in Fig. 2. The analysis of the distribution of turbulent shear stresses shows that the adaptation of $\bar{\tau}_t$ lasts longer than the adaptation of velocity profiles and does not stop within the considered range of curvature parameters.

Figure 3 shows the distribution of the coefficient of friction. Curve 1 corresponds to the case of $\delta_0 = 0$. It is seen that the change in the coefficient of friction is nonlinear. Curves 2-7 indicate the distribution of the coefficient of friction for the case of $\delta_0 \neq 0$ at different values of $\delta_0^{**}/R_w$. It is seen that the curves gradually approach curve 1 and then one curve corresponds to different cases. The character of this change on the adaptation and initial portions differs. On the first portion, the change in the coefficient of friction is caused by two factors: deformation of the velocity profile and the curvature $(\delta^{**}/R_w)$. On the second portion, where the velocity profile is constant, the decrease in the coefficient of friction is due only to the effect of curvature. The values of $\delta_a^{**}/R_w$ at the intersection points of curves characterize the cancellation of adaptation of the boundary layer and the beginning of the main portion where the stabilized curvilinear boundary layer exists.

As a result of generalization of data on the coefficient of friction, we obtained the dependence of the parameter $\delta_a^{**}/R_w$ on the initial parameter of curvature $\delta_0^{**}/R_w$ given in Fig. 4. This dependence is approximated by the expression

$$\frac{\delta_a^{**}}{R_w} = 0.0035 + 2.69 \left(\frac{\delta_0^{**}}{R_w}\right)^{0.92} . \tag{2}$$

The value $\delta_a^{**}/R_w = 0.0035$ at $\delta_0^{**}/R_w = 0$ determined by extrapolation corresponds to the value obtained by the analysis of the adaptation of velocity profiles. Thus, we may conclude that adaptation of the coefficient of friction cancels at the same values of $\delta_a^{**}/R_w$ as adaptation of velocity profiles.

Having multiplied both sides of expression (2) by $U_0/\nu$, we obtain

$$\mathrm{Re}_a^{**} = \mathrm{Re}_0 \left[0.0035 + 2.69 \left(\frac{\delta_0^{**}}{R_w}\right)^{0.92}\right] , \tag{3}$$

where $\mathrm{Re}_0 = U_0 R_w/\nu$.

Expressions (2), (3) make it possible to determine the parameters $\delta_a^{**}/R_w$ and $\mathrm{Re}_a^{**}$ characterizing the cancellation of adaptation of the turbulent boundary layer on transition from the plane to the convex surface within the range $\delta_a^{**}/R_w = 0-0.0054$.

In practical applications, coefficients of friction on the adaptation portion can be calculated by the formula [2]

$$(C_f/C_{f0}) = (1 + 1000\,(\delta^{**} - \delta_0^{**})/R_w)^{-0.31}\,, \tag{4}$$

and on the main portion by

$$(C_f/C_{f0}) = (1 + 1000\,(\delta^{**}/R_w))^{-0.3}\,. \tag{5}$$

Expressions (3)-(5) can also be used in the solution of differential and integral equations of the boundary layer on the convex surface.

## NOTATION

$u$, $v$, longitudinal and transverse components of velocity; $x$, $y$, longitudinal and transverse coordinates; $\tau$, shear stresses ($\tau_v$, viscous); $p$, static pressure; $p_e$, static pressure in the outer flow; $\rho$, density; $\varepsilon$, turbulent kinematic viscosity; $\varepsilon_0$, eddy viscosity on plane surface; $R_w$, surface curvature radius; $\delta$, boundary-layer thickness; $\delta_0$, initial boundary-layer thickness; $u_p$, potential flow velocity; $U_{pw}$, velocity of potential flow on a wall; $u_{p\delta}$, velocity of potential flow on the outer edge of the boundary layer; $R_{ef}$, effective radius of curvature; $\delta^{**}$, momentum-loss thickness; $\delta^{**}/R_w$, curvature parameter; $\delta_0^{**}/R_w$, initial curvature parameter; $\delta_a^{**}/R_w$, curvature parameter for cancellation of adaptation; $\bar{\tau}_t$, turbulent shear stresses; $\mathrm{Re}^{**}$, Reynolds number determined by momentum-loss thickness; $\mathrm{Re}_a^{**}$, number $\mathrm{Re}^{**}$ corresponding to cancellation of adaptation; $U_0$, outer flow velocity; $\mathrm{Re}_0$, Re at entrance; $C_f$, coefficient of friction; $C_{f0}$, coefficient of friction on plane surface.

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